Are we close to multi-scale simultaneous (and robust) seismic imaging?

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Scale separation in seismic imaging

From Jon Claerbout’s “Imaging the Earth Interior”
Gap is closing…

Accuracy

100%

Tomographic velocities

Low-frequency & long-offset data

Velocity

Reflectivity

$log \frac{vk}{2\pi}, \text{ Hz}$
We want to close the gap!
2014 SEG FWI Blind Test (3-35 Hz)

WET+FWI velocity

TFWI velocity

Presented at SEG 2014 Workshop
2014 SEG FWI Blind Test (3-35 Hz)

Image with FWI velocity    Image with TFWI velocity

Presented at SEG 2014 Workshop
FWI vs. TFWI

**FWI**

\[ J_{\text{FWI}}(v) = \frac{1}{2} \| \mathcal{L}(v) - d \|^2 \]

**TFWI**

\[ J_{\text{TFWI}}(\tilde{v}) = \frac{1}{2} \| \tilde{\mathcal{L}}(\tilde{v}) - d \|^2 \pm \varepsilon \| \mathcal{F}(\tilde{v}) \|^2 \]

\( J \) is the objective function to optimize, \( \mathcal{L} \) is non-linear modeling operator, \( v \) is velocity model, \( d \) are data.

\( \tilde{\mathcal{L}}(\tilde{v}) \) is the extended modeling operator, \( \tilde{v} \) is the extended velocity, e.g. \( \tilde{v}(\tau) \), \( \mathcal{F}(\tilde{v}) \) measures focusing of \( \tilde{v} \), e.g. \( \| \tau \tilde{v} \|^2 \).

Symes, 2008 (Geophysical Prospecting)
Biondi and Almomin, 2014 (Geophysics)
Derivation of first-order Born scattering

Full non-linear scattering

\[
\left[ \partial_{tt} - v_0^2 \nabla^2 \right] P_0 = f
\]
\[
\left[ \partial_{tt} - v_0^2 \nabla^2 \right] \delta P = \delta v^2 \nabla^2 \left( P_0 + \delta P \right)
\]

\( v_0 \): Background velocity

\( P_0 \): Background wavefield

\( \delta v \): Velocity perturbation

\( \delta P \): Scattered wavefield

\( f \): Source function

First-order Born scattering

\[
\left[ \partial_{tt} - v_0^2 \nabla^2 \right] P_0 = f
\]
\[
\left[ \partial_{tt} - v_0^2 \nabla^2 \right] \hat{P} = \delta v^2 \nabla^2 P_0
\]

\( \hat{P} \): Born scattered wavefield
Transmission experiment ($t=1.4$ s)
Limitations of Born linearization

Wavefield and data residuals by full non-linear scattering

Wavefield and data residuals by first-order Born scattering
Linearized $\tau$ extension $\tilde{\mathcal{L}}(\tilde{\nu}) = \mathcal{L}(\nu_0) + \tilde{\mathcal{L}} \delta \tilde{\nu}^2(\tau)$

Full non-linear scattering

\[
\begin{align*}
[\partial_{tt} - v_0^2 \nabla^2] P_0 &= f \\
[\partial_{tt} - v_0^2 \nabla^2] \delta P &= \delta v^2 \nabla^2 (P_0 + \delta P)
\end{align*}
\]

$v_0$: Background velocity
$P_0$: Background wavefield
$\delta v$: Velocity perturbation
$\delta P$: Scattered wavefield
$f$: Source function

Linearized $\tau$ extension

\[
\begin{align*}
[\partial_{tt} - v_0^2 \nabla^2] P_0 &= f \\
[\partial_{tt} - v_0^2 \nabla^2] \delta \tilde{P} &= \delta \tilde{v}(\tau)^2 \ast \nabla^2 P_0
\end{align*}
\]

$\delta \tilde{v}(\tau)$: Extended-velocity perturbation
$\delta \tilde{P}$: New scattered wavefield
Linearized $\tau$ extension $\tilde{\mathcal{L}}(\tilde{v}) = \mathcal{L}(v_0) + \tilde{\mathcal{L}} \delta \tilde{v}^2(\tau)$

**Full non-linear scattering**

\[
\left[ \partial_{tt} - v_0^2 \nabla^2 \right] P_0 = f
\]

\[
\left[ \partial_{tt} - v_0^2 \nabla^2 \right] \delta P = \delta v^2 \nabla^2 (P_0 + \delta P)
\]

$v_0$: Background velocity

$P_0$: Background wavefield

$\delta v$: Velocity perturbation

$\delta P$: Scattered wavefield

$f$: Source function

**Linearized $\tau$ extension**

\[
\left[ \partial_{tt} - v_0^2 \nabla^2 \right] P_0 = f
\]

\[
\left[ \partial_{tt} - v_0^2 \nabla^2 \right] \delta \tilde{P} = \delta \tilde{v}(\tau)^2 \overset{*}{\nabla^2 P_0}
\]

$\delta \tilde{v}(\tau)$: Extended-velocity perturbation

$\delta \tilde{P}$: New scattered wavefield
Beyond Born – $\Delta \tilde{\nu}(\tau) = \tilde{L}' \Delta d$

Wavefield computed by full non-linear scattering

Wavefield by linearized $\tau$ extension with $\Delta \tilde{\nu}(\tau) = \tilde{L}' \Delta d$
Beyond Born – $\Delta \tilde{\nu}(\tau) = \tilde{L}' \Delta d$

Wavefield computed by full non-linear scattering

Wavefield by linearized $\tau$ extension with $\Delta \tilde{\nu}(\tau) = \tilde{L}' \Delta d$
Beyond Born $- \Delta \tilde{\nu}(\tau) = \tilde{L}' \Delta d$

Data residuals computed by full non-linear scattering

Data residuals by linearized $\tau$ extension with $\Delta \tilde{\nu}(\tau) = \tilde{L}' \Delta d$
Beyond Born $- \Delta \tilde{\nu} (\tau) = \tilde{\mathcal{L}}' \Delta d$

Data residuals computed by full non-linear scattering

$\tilde{\mathcal{L}}(v) \tilde{\mathcal{L}}'(v) \left[ \mathcal{L}(v) - d \right] \approx \left[ \mathcal{L}(v) - d \right]$

Data residuals by linearized $\tau$ extension with $\Delta \tilde{\nu} (\tau) = \tilde{\mathcal{L}}' \Delta d$
Practical TFWI algorithm

\[
\tilde{J}(v, \delta \tilde{v}) = \left\| \mathcal{L}(v) + \tilde{L}(v) \delta \tilde{v} - d \right\|^2_2 + \varepsilon \left\| F \delta \tilde{v} \right\|^2_2
\]

Biondi and Almomin, 2014 (Geophysics)
Practical TFWI algorithm

$$\tilde{J}(v, \delta \tilde{v}) = \| \mathcal{L}(v) + \tilde{L}(v) \delta \tilde{v} - d \|^2_2 + \varepsilon \| F \delta \tilde{v} \|^2_2$$

Inner loop – Estimate $v$ and $\delta \tilde{v}$ with fixed $v$

Inner loop – Scale mixing between $v$ and $\delta \tilde{v}$

Outer loop – Update $v$ with low-pass of $\delta \tilde{v} + v$

Biondi and Almomin, 2014 (Geophysics)
IO-Jansz

NW Australia

Conventional streamer data

Ali Almomin’s thesis – SEP 164
Initial velocity
Image with initial velocity

Ali Almomin’s thesis – SEP 164
Image with TFWI velocity
CIGs with initial velocity

Ali Almomin's thesis – SEP 164
CIGs with TFWI velocity
“Variable Projection” algorithm

\[
\tilde{J}(v, \delta \tilde{v}) = \left\| \mathcal{L}(v) + \tilde{L}(v) \delta \tilde{v} - d \right\|^2_2 + \varepsilon \left\| F \delta \tilde{v} \right\|^2_2
\]

Inner loop – Estimate \( \delta \tilde{v} \) with fixed \( v \) as:

\[
\delta \tilde{v} = \left[ \tilde{L}'(v) \tilde{L}(v) + \varepsilon F'F \right]^{-1} \left[ \mathcal{L}(v) - d \right]
\]

Outer loop – Update \( v \) by minimizing:

\[
\hat{J}(v) = \left\| \mathcal{L}(v) + \hat{L}(v) \delta \tilde{v} - d \right\|^2_2 + \varepsilon \left\| F \delta \tilde{v} \right\|^2_2
\]

Variable Projection algorithm: Golub and Pereyra, 1973 (SIAM)
Variable Projection for WI: van Leeuwen and Moulder, 2009 (EAGE), Huang and Symes, 2015 (SEG), …
Preconditioned “Variable Projection”

\[ \tilde{J}(\mathbf{v}, \delta \tilde{\mathbf{v}}) = \| \mathcal{L}(\mathbf{v}) + \tilde{\mathbf{L}}(\mathbf{v}) \delta \tilde{\mathbf{v}} - \mathbf{d} \|_2^2 + \varepsilon \| F \delta \tilde{\mathbf{v}} \|_2^2 \]

**Inner loop – Estimate \( \delta \tilde{\mathbf{v}} \) with fixed \( \mathbf{v} \) as:**

\[ \delta \tilde{\mathbf{v}} = \left[ \tilde{\mathbf{L}}_p(\mathbf{v}) \tilde{\mathbf{L}}_p(\mathbf{v}) + \varepsilon \mathbf{I} \right]^{-1} \left[ \mathcal{L}(\mathbf{v}) - \mathbf{d} \right] \]

**Outer loop – Update \( \mathbf{v} \) by minimizing:**

\[ \hat{J}(\mathbf{v}) = \| \mathcal{L}(\mathbf{v}) + \tilde{\mathbf{L}}(\mathbf{v}) \delta \tilde{\mathbf{v}} - \mathbf{d} \|_2^2 + \varepsilon \| F \delta \tilde{\mathbf{v}} \|_2^2 \]

Related to M. Clapp’s thesis – SEP 122

G. Barnier, E. Biondi, and B. Biondi, 2017 (SEP 170)
Gradient with respect to $V$

\[
\hat{J}(v) = \left\| \mathcal{L}(v) + \mathcal{L}(v) \delta v - d \right\|^2_2 + \epsilon \left\| F \delta v \right\|^2_2
\]

\[
\nabla \hat{J} = \left[ L'(v) + T'(v) \right] \left[ \mathcal{L}(v) + \mathcal{L}(v) \delta v - d \right]
\]

where: $L(v)$ is the linearization of $\mathcal{L}$ w.r.t. $v$,

$T(v)$ is the linearization of $\mathcal{L}$ w.r.t. $v$.
Gradient with respect to $V$

$$\hat{J}(v) = \left\| \mathcal{L}(v) + \tilde{L}(v) \delta \tilde{v} - d \right\|_2^2 + \epsilon \left\| F \delta \tilde{v} \right\|_2^2$$

Migration? Tomography?

$$\nabla \hat{J} = \left[ L'(v) + T'(v) \right] \left[ \mathcal{L}(v) + \tilde{L}(v) \delta \tilde{v} - d \right]$$

where: $L(v)$ is the linearization of $\mathcal{L}$ w.r.t. $v$, $T(v)$ is the linearization of $\tilde{L}$ w.r.t. $v$. 

G. Barnier, E. Biondi and B. Biondi, 2017 (SEP 170)
Without vs. With preconditioning: Mora’s model

**True model**

**(True – Initial) model**
Without vs. With preconditioning: Convergence

No preconditioning of $F$

With preconditioning of $F$
\[ T'(v) \left[ \mathcal{L}(v) + \tilde{L}(v) \delta \tilde{v} - d \right] - 20 \text{ iterat. of inner loop} \]

No preconditioning of \( F \)  

With preconditioning of \( F \)
\[ T'(v) \left[ \mathcal{L}(v) + \mathcal{L}(v) \hat{\delta}v - d \right] - 40 \text{ iterat. of inner loop} \]

No preconditioning of \( F \)  
With preconditioning of \( F \)
$L'(v) \left[ \mathcal{L}(v) + \hat{L}(v) \delta \hat{v} - d \right] - 20 \text{ iterat. of inner loop}$

No preconditioning of $F$

With preconditioning of $F$
\[ L'(v) \left[ \mathcal{L}(v) + \hat{L}(v) \delta \hat{v} - d \right] - 40 \text{ iterat. of inner loop} \]
$\left[ L'(v) + T'(v) \right] \left[ \mathcal{L}(v) + \tilde{L}(v) \delta \tilde{v} - d \right] : 20 \text{ iterations}$

No preconditioning of $F$  

With preconditioning of $F$
\[
\left[ L'(v) + T'(v) \right] \left[ L(v) + \tilde{L}(v) \delta v - d \right] : 40 \text{ iterations}
\]
Without preconditioning of $F$

Inner loop: 40 iterations – Outer loop: 2 iterations

(Final – Initial) model
(True – Initial) model
With preconditioning of F

Inner loop: 40 iterations – Outer loop: 2 iterations

(Final – Initial) model  (True – Initial) model
Conclusions

• We (as a community) are making progress towards multi-scale simultaneous (and robust) seismic imaging.
• Waveform inversion with (time) model extension have encouraging convergence properties, even in 3D field data problems.
• A “Preconditioned Variable Projection” algorithms is a promising new direction that may speed-up convergence and reduce computational cost.
Acknowledgments

• Ali Almomin (Aramco) for doing some of the work, and providing slides.

• Chevron for IO-Jansz data.

• Chevron for the SEG 2014 blind-test data.

• Stanford Exploration Project affiliate members for financial support

• Chevron for financial support of Stanford Center of Research Excellence.

• Stanford Center for Computational Earth and Environmental Science for computational support.
Beyond Born – Extended velocity

Horizontal section across anomaly

\[ \delta \tilde{v}^2(\tau) \]
Beyond Born – Extended velocity

Vertical section across anomaly
Beyond Born – Data residuals

Wavefield and data residuals by full non-linear scattering

Wavefield and data residuals by linearized $\tau$ extension