FWI for Elastic Media - Macrovelocity reconstruction

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Outline

1. Gullfaks model.
2. DSR-FWI modified formulation (MBTT).
3. Cross-talk for P- and S-wave interfaces.
4. Numerical experiments with Gullfaks model.
5. Conclusions and road map.
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In our numerical experiments we used 2D elastic version of Gullfaks oilfield on the Norwegian continental shelf, but without water on the top of the model.
Gullfaks model

Forward map: mathematical model
To describe isotropic elastic wave propagation we use velocity-stress formulation as first order system of PDE

\[
\begin{align*}
      i \omega M(x, z) \vec{U} - P \frac{\partial \vec{U}}{\partial x} - Q \frac{\partial \vec{U}}{\partial z} &= \vec{f} \\
      \vec{U} &= (U_x, U_z, \sigma_{xx}, \sigma_{zz}, \sigma_{xz})
\end{align*}
\]

\[
M = \begin{pmatrix} \rho I_{2x2} & 0 \\ 0 & S_{3x3} \end{pmatrix}, \quad P = \begin{pmatrix} 0 & A \\ A^T & 0 \end{pmatrix}, \quad Q = \begin{pmatrix} 0 & B \\ B^T & 0 \end{pmatrix},
\]

\[
A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},
\]

\[
S = \begin{pmatrix} a & -b & 0 \\ -b & a & 0 \\ 0 & 0 & c \end{pmatrix}, \quad a = \frac{\lambda + 2\mu}{4\mu(\lambda + \mu)}, \quad b = \frac{\lambda}{4\mu(\lambda + \mu)}, \quad c = 1/\mu.
\]

S – compliance tensor
Gullfaks model

**Forward map: boundary conditions**

At this stage we cancel water layer at the top, fill it with elastic medium and introduce PML on all sides of the rectangle. Next we do finite-difference approximation of this boundary value problems and on this base introduce nonlinear operator transforming distribution of P-, S-wave velocities, density and source (position and function) to the data recorded on the given acquisition system:

\[ F : m \rightarrow \tilde{U} \]
Gullfaks model

ABC (PML)
Gullfaks model: acquisition

• Frequency range: 5 – 25 Hz, 21 uniformly sampled frequencies;
• 39 vertical sources at the depth 20 m, sampled with 100 m from 100 m to 3900 m;
• 198 receivers at the same depth sampled with 20 m from 20 m to 3980 m.
Gullfaks elastic model: results
Gullfaks model

Initial approximation: 1D model
Gullfaks elastic model

Inversion (Vp)
Gullfaks elastic model

Inversion (Vs)
Gullfaks elastic model
Inversion (Vp/Vs)
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DSR-FWI modified formulation

$L_2$ FWI misfit function (Tarantola et al., 1980-th)

$$E(m) = \frac{1}{2} \left\| d^{obs} - F(m) \right\|_D^2$$

Standard non-linear $L_2$ formulation of FWI:

$$E(m) \rightarrow \text{min}$$
Least Squares FWI: macrovelocity reconstruction (7 – 25 Hz)
Conventional Least Squares FWI: propagator?

From: (Gauthier, Virieux and Tarantola; 1986)
Can we change formulation of FWI to enhance sensitivity to propagator without low time frequencies?
DSR-FWI modified formulation: propagator / reflector decomposition

- The model \( (V_p(x,z), V_s(x,z)) \) is decomposed to a smooth propagator and rough spatial reflectivity:
  \[
  V_p(x,z) = PV_p(x,z) + RV_p(x,z) \\
  V_s(x,z) = PV_s(x,z) + RV_s(x,z)
  \]

- Smooth propagators \( PV_p(x,z) \) and \( PV_s(x,z) \) do not (almost) produce return of the seismic energy back to the acquisition, but govern propagation time;

- Rough reflector \( RV_p(x,z) \) and \( RV_s(x,z) \) (almost) do not change propagation time, but change direction of seismic energy and send it back to acquisition.
DSR-FWI modified formulation: propagator / reflector decomposition (naive)
DSR-FWI modified formulation: propagator / reflector decomposition

For known propagator $P$, we parameterize spatial reflectivity $R$ by data space reflectivity ($DSR$) $s$ via some imaging operator

$$V_{p,s} = PV_{p,s} + RV_{p,s} = PV_{p,s} + MV_{p,s}(PV_{p,s}) < s >$$

Currently we use a weighted/true amplitude migration operator $MV_{p,s}(PV_{p},PV_{s})$ derived on the base of the adjoint state techniques:

$$MV_{S,P}(PV_{p},PV_{p}) < s > = W_{S,P}^{\circ} Re \left\{ \left( \frac{\partial F}{\partial V_{S,P}} \right)^{*} < s > \right\}$$
DSR-FWI modified formulation

On this base we reformulate the data misfit functional:

\[ E(V_p, V_s) = \frac{1}{2} \left\| d^{obs} - F(V_p, V_s) \right\|^2_D, \]

as follows:

\[ \tilde{E}(PV_p, PV_s; RV_p, RV_s) = \]

\[ = \frac{1}{2} \left\| d^{obs} - F(PV_p + M_p(PV_p, PV_s) < s >, PV_s + M_s(PV_p, PV_s) < s >) \right\|^2_D \]
The main difference with standard $L_2$ Full Waveform Inversion:

- propagators are searched for in the **model space**;
- reflectivity is searched for in the **data space**.

$$\tilde{E}( PV_p, PV_s; M_p( PV_p, PV_s ) < s >, M_s( PV_p, PV_s ) < s > ) \rightarrow \min_{PV_p, PV_s, s}$$
DSR-FWI modified formulation: Frechet derivative

Linearized standard FWI in the vicinity of some model $m_0$:

$$\frac{\delta F}{\delta m}(m_0) \langle \delta m \rangle = \delta d$$

Linearized with respect to propagator in the vicinity of the same model $m_0$:

$$\frac{\delta F}{\delta m}(m_0 + M(m_0) \langle s_{true} \rangle) \langle \delta p + W \circ \left( (\delta^2 F(m_0) / \delta m^2) \langle ., \delta p \rangle \right)^* \langle s_{true} \rangle = \delta d$$

If data space reflectivity $s_{true}$ is equal to zero, these derivatives are identical!
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DSR-FWI modified formulation: cross-talk between Vp and Vs interfaces
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Strategy of non-linear inversion

The workflow of non-linear DSR-FWI (MBTT) modified inversion:

Stage 1: minimization with respect to data space reflectivity $s$

Stage 2: minimization with respect to propagator $p$

Stage 3: minimization with respect to data space reflectivity $s$

......... etc.
Numerical experiments: 2D elastic Gullfaks model

P-propagator recovery
Numerical experiments: 2D elastic Gullfaks model

Vp recovery
Numerical experiments: 2D elastic Gullfaks model

S-propagator recovery
Numerical experiments: 2D elastic Gullfaks model

Vs recovery
Numerical experiments: 2D elastic Gullfaks model

Vp/Vs recovery
Numerical experiments: 2D elastic Gullfaks model

Data comparison
Numerical experiments: 2D elastic Gullfaks model

Data comparison

[Graphs showing Uz true, Uz start, and Uz recovered data comparison]
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Conclusions

• Modification of multicomponent (elastic) FWI formulation on the base of Data Space Reflectivity is introduced and implemented;

• Numerical experiments prove advantages of this formulation for both $V_p$ and $V_s$ propagator reconstruction without of unreasonable low time frequencies, but at the moment without of free surface (no ground roll).
Road map

• Application of more effective minimization techniques in 2D elastic statement (Gauss-Newton, modified Newton,.....)
• 2D Elastic formulation for onshore seismic acquisition with free surface (ground roll consideration)
• 3D offshore with the base of 3D elastic direct solver (HSS): isotropy, anisotropy, viscoanisotropy...
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THANK YOU FOR ATTENTION
Questions?